Hokkaido Mathematical Journal Vol. 19 (1990) p. 89-101

On Coxeter functors over tensor rings with duality conditions

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(Received August 24, 1988)

1. Introduction and preliminaries

Two pairs of functors play an outstanding role in the representation theory of finite-dimensional tensor algebras: The Coxeter functors C^{+} and C^{-} and the functors DTr and TrD, the importance of which has been discovered by Auslander and Reiten (see [5] resp. [1]; as usual Tr denotes the Auslander-Bridger transpose and D the duality with respect to the ground field). Brenner and Butler [4] and, independently, Gabriel [6] have proved the remarkable fact that there is an equivalence T of a very simple form such that C^{+} and $DTrT$ resp. C^{-} and $TDTr$ are isomorphic. The validity of a result of this sort for the larger class of artinian tensor rings with duality conditions has been conjectured by Auslander, Platzeck and Reiten [2] when they noticed that these rings possess a canonical selfduality D. In the present article this conjecture is confirmed. Actually we intend to show that Gabriel's proof can be adopted with certain modifications.

For the convenience of the reader we sum up some definitions and simple facts concerning tensor rings with duality conditions and their modules. Since it is more suitable for our purpose we prefer the equivalent language of modulations of quivers and their representations [5]. However, first let us agree upon some conventions. For modules M, N over some ring S we shall write (M, N) instead of $\mathrm{Hom}_S(M, N)$; furthermore we use the abbreviations $M^{*}=(M_S, S_S)$ resp. ${}^{*}M=({}_SM, {}_SS)$ for a right resp. left S-module M. We shall place maps of left modules to the right of the argument and maps of right or bimodules to the left; accordingly the composition of maps is written. In the situation $M_S, {}_SN_T, P_T$ the canonical isomorphism $(M\otimes_S N_T, P_T)\longrightarrow(M_S, (N_T, P_T)_S)$ is denoted by $f\longmapsto\hat{f}$.

In this paper we assume that Γ is a finite connected quiver without cycles and multiple arrows. The set of vertices resp. arrows of Γ is denoted by Γ_0 resp. Γ_1 and the domain resp. range of some arrow α by $d\alpha$ resp. $r\alpha$. Furthermore we assume that for each $x\in\Gamma_0$ we have a