On serial quasi-hereditary rings

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Dedicated to Professor Tosiro Tsuzuku on his 60th birthday

In their Carleton Lecture Note [3], V. Dlab and C. M. Ringel studied the quasi-hereditary rings initiated by E. Cline, B. Parshall and L. Scott [1] and applied to the representation theory of algebras. The quasi-hereditary algebras generalize the hereditary algebras and have the finite global dimension. But not all algebras of finite global dimension are quasi-hereditary, though the algebras of global dimension 2 are quasi-hereditary [3]. In fact, they showed an example of a non-quasi-hereditary algebra of global dimension 4 and dominant dimension ≥ 2. Taking account of these facts, Dlab posed a question in [2] whether the algebras of global dimensaion 3 are quasi-hereditary. The aim of this note is to show that serial Artinian rings (= Nakayama rings) of global dimension 3 are quasi-hereditary, and to answer in the negative to his question by showing an example of an algebra, without any heredity ideals, whose global dimension and dominant dimension are three.[*] In the first two sections, we shall give two remarks concerning the refinement of heredity chains and Morita invariance of the quasi-hereditarity of rings. In the final section, some examples will be given and some problem, which is naturally arised from those examples, will be discussed.

Throughout this note, all rings are semi-primary and, unless specified otherwise, all modules are right modules. Denoted by add M we understand the category of modules which are isomorphic to direct summands of direct sums of copies of M. For a given ring A, the Jacobson radical will be denoted by N.

1. Refinement of heredity chains

In this section we shall show that all heredity chains are refined to heredity chains with the same length as the number of simple modules.

We first recall from [3] the definition of heredity chains. Let A be a

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[*] Our example was announced in a lecture of Dlab at the Conference of Representation Theory of Algebras held at the Banach Center (Warsaw, April, 1988).