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On bicommutators of modules over H-separable extension rings II

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This paper is a continuation of the author's previous paper [13], and is devoted to its application. Therefore we will use the same notation as [13]. In § 1 we will give some supplements to [13], and the results of § 1 and [13] will be applied to Azumaya algebras in § 2. Azumaya algebra is the model of the H-separable extension. In general, when we dealt with modules over Azumaya algebras, they were almost limited to be finitely generated projective. Here we will deal with general modules and show that, if A is an Azumaya R-algebra, then for any left A-module M $A^{*}=\mathrm{Bic}({}_{A}M)$ is an Azumaya R^{*}-algebra, with $R^{*}=\mathrm{Bic}({}_{R}M)$ commutative, and we have $R^{*}\cap\iota(A)=\iota(R)$, $A^{*}\cong A\otimes_{R}R^{*}$, where ι is the canonical map of A to A^{*} (Theorem 2). Furthermore if M is A-faithful, there exist mutually inverse 1-1-correspondences between the class $\mathscr{S}$ of intermediate rings between R and R^{*} and the class $\mathscr{T}$ of intermediate rings between A and A^{*}, given by $T\rightarrow AT$, and $S\rightarrow S\cap R^{*}$, for $T\varepsilon\mathscr{T}$ and $S\varepsilon\mathscr{S}$. In addition, every ring S belonging to $\mathscr{S}$ is an Azumaya $(S\cap R^{*})$-algebra (Proposition 3). In § 3 we deal with the H-separable extension of strongly primitive rings. Strongly primitive ring is the one which has a faithful minimal left, or equivalently right, ideal. Suppose that A and B are strongly primitive rings, and let M and $\mathfrak{m}$ be faithful minimal left ideals of A and B, respectively. If A is left B-finitely generated projective and H-separable over B, then $B^{*}=\mathrm{Bic}({}_{B}M)\cong\mathrm{Bic}({}_{B}\mathfrak{m})$, and $A^{*}=\mathrm{Bic}({}_{A}M)$ is an H-separable extension of B^{*} (See Theorom 3. 3 [12]). In this paper we will show under the same condition as above that $A^{*}=\bar{A}B^{*}=B^{*}\bar{A}$, $B^{*}\cap\bar{A}=\bar{B}$, where $\bar{A}=\iota(A)$ and $\bar{B}=\iota(B)$, and for any strongly primitive subring S of A such that $B\subset S$ and A is left S-projective, A^{*} is H-separable over S^{*}, and $S^{*}=\mathrm{Bic}({}_{S}M)$ is also a full linear ring (Theorem 4).

1. In this section A will always be a ring with the identity 1 and B a subring of A containing 1. C is the center of A and $D=V_{A}(B)$, the centralizer of B in A. For a left A-module M we write $A^{*}=\mathrm{Bic}({}_{A}M)$, the bicommutator of ${}_{A}M$, $B^{*}=\mathrm{Bic}({}_{B}M)$, $D^{*}=V_{A*}(B^{*})$ and $C^{*}=V_{A*}(A^{*})$,