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A unit group in a character ring of an alternating group

Dedicated to Professor Kazuhiko Hirata on his 60th birthday

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1. Introduction

Throughout this paper, G denotes always a finite group, Z a ring of rational integers, Q a rational field and C a complex field. Let $\{x_1$(a principal character),$\cdots$,$x_h\}$ be the set of all irreducible C-characters of G. We denote this set by Irr(G). Let us set

$$R(G)=\{\sum_{i=1}^{h} a_i x_i | a_i \in Z\}$$

That is, $R(G)$ is the set of generalized characters of G. It is well known that $R(G)$ forms a commutative ring with an identity element x_1. We call $R(G)$ a character ring of G.

Let ζ be a primitive $|G|$-th root of unity and let $K=Q(\zeta)$ be the smallest subfield of C containing Q and ζ. We denote by A the ring of algebraic integers in K. In the paper of [9], we have proved the following theorem and corollary.

THEOREM 1.1. *Any unit of finite order in* $A\otimes_Z R(G)$ *has the form* $\varepsilon\chi$ *for some linear character* χ *of* G *and some unit* ε *in* A.

COROLLARY 1.2. *Any unit of finite order in* $R(G)$ *has the form* $\pm\chi$ *for some linear character* χ *of* G.

We denote by $U(R(G))$ a unit group of $R(G)$. In section 2, we shall prove that $U(R(G))$ is finitely generated. Hence a factor group $U(R(G))/U_f(R(G))$ is a free abelian group of finite rank, where $U_f(R(G))$ is the group which consists of units of finite order in $R(G)$.

In this paper, we intend to compute the rank of $U(R(A_n))/U_f(R(A_n))$, where A_n is an alternating group on n symbols.

2. Preliminaries

We first show that $U(R(G))$ is finitely generated.

THEOREM 2.1. *For a finite group* G, $U(R(G))$ *is finitely generated.*