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Lifting Fourier-Stieltjes transforms and transferring cocycles

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Abstract

We exhibit a class of linear liftings of Fourier-Stieltjes transforms defined on a closed subgroup of a locally compact Abelian group to Fourier-Stieltjes transforms defined on the whole group. Using these liftings, we establish a result about unitary representations associated with cocycles on compact Abelian groups with dense action.

0. Introduction

Let G be a locally compact Abelian group and $\widehat{G}$ be the dual group of G. Let $A(G)$ be the space of Fourier transforms of Haar-integrable functions on $\widehat{G}$, $B(G)$ be the space of Fourier transforms of complex finite regular Borel measures on $\widehat{G}$, $B_{+}^{1}(G)$ be the set of Fourier transforms of regular Borel probability measures on $\widehat{G}$, and $B_{s}(G)$ be the space of Fourier transforms of finite regular Borel measures on $\widehat{G}$ singular with respect to Haar measure. Let G_0 be a closed subgroup of G and R be the operator of the restriction to G_0 of functions defined on G. A well-known elementary result states that $R(A(G))=A(G_0)$ and $R(B(G))=B(G_0)$ (cf. [13, Theorems 2.7.2 and 2.7.4]). J. Inoue [10] constructed a linear isometry I from $B(G_0)$ into $B(G)$, carrying $A(G_0)$ in $A(G)$, $B_{+}^{1}(G_0)$ in $B_{+}^{1}(G)$, and $B_s(G_0)$ in $B_s(G)$, such that RI is the identity on $B(G_0)$ and, for each $\psi\in B(G_0)$, the support of $I\psi$ is contained in the set of all elements of the form $x+y$ with x in the support of ψ and y in any given neighbourhood of 0 in G. Inoue's construction, relying on a subtle reduction to the case in which G_0 is discrete and in which such an isometry can be expressed by a simple formula (cf. [9, Theorem A. 7. 1]) is fairly complicated and leads to a rather non-transparent formula for I. In this paper, we reveal a class of isometries with properties as above, which have a strikingly simple form. Taking advantage of the special shape of these isometries, we establish a result about transferring cocycles from closed subgroups of compact Abelian groups with dense action to the entire groups. The latter result will provide motivation to the proposed approach.