Hokkaido Mathematical Journal Vol. 20 (1991) p. 465-475

Existence results for singular elliptic equations

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(Received January 16, 1990, Revised June 15, 1990)

The purpose of this paper is to study the existence of positive solutions in $\boldsymbol{R}_n$ of the singular elliptic equation

$$(1) \qquad Lu=-\sum_{i,j=1}^{n} D_i(a_{ij}(x)D_ju)+c(x)u=g(x, u),$$

where the nonlinearity g is defined on $\boldsymbol{R}_n\times(0,\infty)$. Solutions of (1), which are defined on $\boldsymbol{R}_n$, are called entire solutions. The precise conditions on g, to be formulated later, show that equation (1) is a natural extension of the following equation

$$(1') \qquad -\Delta u=f(x)u^{-\gamma} \text{ in } \boldsymbol{R}_n,$$

where $\gamma>0$ is a constant. The equation (1′) is called in the existing literature the Lane-Emden-Fowler equation and arises in the boundary-layer theory of viscous fluids (see [4], [5], [6], [8] and the references given there). In papers [4] and [8] it is assumed that $f(x)$ depends "almost" radially on x in the sense that

$$c_1p(|x|)\leqq f(x)\leqq c_2p(|x|),$$

where $c_1>0$ and $c_2>0$ are constants and $p(|x|)$ is a positive function satisfying some integrability condition. The existence results are then obtained using the method of sub and supersolutions. In [5] the existence of positive solutions was obtained by replacing (1′) with an equivalent operator equation which can be solved using the Schauder-Tichonov fixed point theorem. In this paper we develop ideas from paper [1], where the existence of weak solutions, in the case $g(x, u)=f(x)u^{-\gamma}$, $0<\gamma<\infty$, has been considered. Here we consider more general nonlinearities g. Our method in based on approximation arguments. We first solve the Dirichlet problem in a bounded domain with zero boundary data. An entire solution is then obtained as a limit of solutions u_m of the Dirichlet problems on Ω_m, with $\{\Omega_m\}$ exhausting $\boldsymbol{R}_n$. The assumptions (g_1) and (g_2) ensure that solutions of the Dirichlet problem in a bounded domain Ω belong to $W_{\mathrm{loc}}^{1,2}(\Omega)\cap C(\bar{\Omega})$. We also point out that under some additional