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Timelike surfaces in Lorentz 3-space with prescribed mean curvature and Gauss map

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A timelike surface M^2 in Lorentz 3-space $\boldsymbol{L}^3$ is a surface which inherits a non-degenerate indefinite metric from the standard metric in $\boldsymbol{L}^3$. A Gauss map can be defined on $\boldsymbol{M}^2$ with values in the unit sphere $S_1^2 \subset \boldsymbol{L}^3$.

We will prove in Theorem 3.1 that the Gauss map and mean curvature of a timelike surface satisfies a system of partial differential equations. As a corollary the Gauss map of a timelike minimal surface is shown to satisfy a simple hyperbolic system. This is the precise analogue of the theorem that the Gauss map of a minimal surface in Euclidean space is a holomorphic map into the Riemann sphere. In the latter case the Cauchy-Riemann equations should be thought of as a simple elliptic system of partial differential equations.

In section 4 we find representations for a timelike surface in terms of its Gauss map and mean curvature. The integrability condition for this formula is a pair of partial differential equations (5.2a, b). In Theorem 6.1 we show that given functions defined on a simply connected surface which satisfy the integrability conditions we can find an isometric immersion with these functions as Gauss map and mean curvature.

Let us also note that in Theorem 4.3 we give a Weierstrass representation for timelike minimal surfaces without flat points. Timelike minimal surfaces have recently been the subjects of several papers [Ma2], [Mi2], [Mi3] and [Mi4].

All of these results are timelike versions of the work of K. Kenmotsu [Ke] for a surface in Euclidean 3-space and K. Akutagawa and S. Nishikawa [A-N] for a spacelike surface in $\boldsymbol{L}^3$, and our debt to these authors is clear. They consider M^2 as a Riemann surface, introducing a complex variable via isothermal coordinates. Thus their results are cast in the language of complex analysis. In the timelike case there is no such natural complex structure on M^2. It is somewhat surprising that the same types of results can still be proven, but this really shows that complex analysis is, for the most part, a useful calculational device in [Ke] and

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