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Kirillov-Kostant theory and Feynman path integrals on coadjoint orbits I

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Introduction

In this paper we compute the examples of the Feynman path integrals on the coadjoint orbits of noncompact Lie groups. We follow the method given by Alekseev, Faddeev and Shatachvili [5], where they constructed all irreducible representations of compact Lie groups. Trying to generalize their results to noncompact case, we encountered several crucial difficulties which we overcame by means of the case by case method. We still do not know any unified method which works for general Lie groups.

Let G be a Lie group and $\mathfrak{g}$ the Lie algebra of G. Fix an element λ of the dual space $\mathfrak{g}^*$ of $\mathfrak{g}$ and choose a polarization $\mathfrak{p}$. Following the Kirillov-Kostant theory, we construct an irreducible unitary representation $\pi_\lambda^{\mathfrak{p}}$ of G on the Hilbert space $\mathscr{H}_\lambda^{\mathfrak{p}}$ of all partially holomorphic sections of the line bundle L_{ξ_λ} associated with the character ξ_λ of the subgroup P corresponding to the polarization $\mathfrak{p}$. We remark that if $\mathfrak{p}$ is real P is a Lie subgroup of G such that the Lie algebra of P coincides with $\mathfrak{p}$ and that if $\mathfrak{p}$ is to tally complex and if the complexification G^{C} of G exists P denotes the complex analytic subgroup of G^{C} corresponding to $\mathfrak{p}$.

Let θ be the canonical 1-form on G [23]. Put $\theta_\lambda=\langle\lambda, \theta\rangle$ and $W=GP$. Taking a suitable coordinate system which gives a local triviality of the principal fiber bundle $W\longrightarrow W/P=G/(G\cap P)$, we choose a "good" 1-form $\alpha_{\mathfrak{p}}$ such that $\theta_\lambda-\alpha_{\mathfrak{p}}$ is an exact form. For any $Y\in\mathfrak{g}$, we put $H_Y(g)=\langle Ad^*(g)\lambda, Y\rangle$, which we call the hamiltonian corresponding to Y. Here $Ad^*(g)$ denotes the coadjoint action of g.

The purpose of this paper is to show by explicitly computable simple examples that if one chooses the above "good" 1-form $\alpha_{\mathfrak{p}}$ the "path integral" computed by using the action $\int_0^T \gamma^*\alpha_{\mathfrak{p}}-H_Y dt$ (where γ runs over a certain set of paths on the coadjoint orbit) and the measure defined by the canonical symplectic structure of the coadjoint orbit gives the representation $\pi_\lambda^{\mathfrak{p}}$ of G on $\mathscr{H}_\lambda^{\mathfrak{p}}$.