Hokkaido Mathematical Journal Vol. 20 (1991) p. 213-239

Pseudo-hermitian symmetric spaces and Siegel domains over nondegenerate cones

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(Received April 26, 1990)

Dedicated to Professor Noboru Tanaka on his sixtieth birthday

Introduction

Korányi-Wolf [13] established a method of realizing a hermitian symmetric space M_0 of noncompact type, equivariantly imbedded in its compact dual M^*, as a Siegel domain, by means of a so-called Cayley transform. The goal of this paper is to develop an analogy of the Korányi-Wolf theory for a certain class of complex affine symmetric spaces, called *simple irreducible pseudo-hermitian symmetric spaces of* K_ε*-type* (For the definition, see 2. 3. Also see 5. 2). It is proved that such a space arises as an open orbit in M^* under the identity component of the holomorphic automorphism group of M_0 (Proposition 3. 7). For our purpose, we introduce the notion of a *Siegel domain over a nondegenerate cone* (§ 1), which is a generalization of a Siegel domain over a positive definite (=self-dual) cone. Contrary to the hermitian symmetric case, not the whole part of a simple irreducible pseudo-hermitian (non-hermitian) symmetric space of K_ε-type but an open dense subset of it is realized as an affine homogeneous Siegel domain over a nondegenerate cone (Theorem 5. 3). This realization might serve the study of the boundary of such a symmetric space imbedded in M^*.

In § 1, the closure structure of a Siegel domain over a nondegenerate cone is given (Theorem 1. 1). In § 2, a signature of roots (Oshima-Sekiguchi [18]) of a semisimple Lie algebra $\mathfrak{g}$ is described in terms of a gradation of $\mathfrak{g}$. Given a real simple Lie algebra $\mathfrak{g}$ of hermitian type, we construct in § 3 all simply connected irreducible pseudo-hermitian symmetric spaces of K_ε-type associated with $\mathfrak{g}$ (Theorem 3. 6). In § 4, we give the graded Lie algebraic approach to the Korányi-Wolf theory. Let $\mathfrak{g}=\sum_{k=-2}^{2}\mathfrak{g}_k$ be a simple graded Lie algebra of hermitian type corresponding to a Siegel domain (The case $\mathfrak{g}_{-1}=\mathfrak{g}_1=(0)$ may occur). We then obtain the orbit decomposition of $\mathfrak{g}_{-2}$ under the adjoint action of the group generated by the Lie algebra $\mathfrak{g}_0$ (Theorem 4. 9). In § 5, we give a list of simple