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On oblique derivative problems for fully nonlinear second-order elliptic PDE's on domains with corners

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§1. Introduction

This note is a sequel to our study [4] of oblique derivative problems for fully nonlinear elliptic PDE's on nonsmooth domains.

Let Ω be a bounded open subset of $\boldsymbol{R}^N$. We assume that Ω may be represented as

$$(1.1)\qquad \Omega=\bigcap_{i\in I}\Omega_i,$$

where I is a finite index set and the Ω_i's are domains of $\boldsymbol{R}^N$ with relatively regular boundary. For $x\in\partial\Omega$ we denote by $I(x)$ the set of those indices i which satisfy $x\in\partial\Omega_i$. Let $\{\gamma_i\}_{i\in I}$ be a set of vector fields on $\boldsymbol{R}^N$ and $\{f_i\}_{i\in I}$ a set of real functions on $\partial\Omega\times\boldsymbol{R}$. We assume that each γ_i is oblique to Ω_i on $\partial\Omega_i$, i. e., $\langle\gamma_i(x), n_i(x)\rangle>0$ for $x\in\partial\Omega_i$, where $n_i(x)$ denotes the outward unit normal vector of Ω_i at x.

We consider the fully nonlinear elliptic PDE

$$(1.2)\qquad F(x, u, Du, D^2u)=0 \text{ in } \Omega,$$

together with the oblique derivative conditions

$$(1.3)\qquad \frac{\partial u}{\partial\gamma_i}+f_i(x, u)=0 \text{ for } x\in\partial\Omega \text{ and } i\in I(x).$$

Here u represents a real unknown function on $\bar{\Omega}$, F is a given real function on $\bar{\Omega}\times\boldsymbol{R}\times\boldsymbol{R}^N\times\boldsymbol{S}^N$, where $\boldsymbol{S}^N$ denotes the space of $N\times N$ real symmetric matrices with the usual ordering, and Du and D^2u denote the gradient and Hessian matrix of u, respectively.

Our basic assumption on F is the degenerate ellipticity. That is, we assume that

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