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On P-Galois extensions of rings of cyclic type

Dedicated to Professor Tosiro Tsuzuku on his 60th birthday

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§1. A relative sequence of homomorphisms P and a P-Galois extension.

Let B be a ring with an identity 1 and A a subring of B with common identity 1 of B. In [6], the author studied on a relative sequence of homomorphisms P of $End(B_A)$ and a P-Galois extension B/A. In this paper we shall study on constructive P-Galois commutative extensions of cyclic type as an application of the works of [6].

For the convenience of readers, we shall summarized notions and several properties of a relative sequence of homomorphisms P and a P-Galois extension. The details and proofs will be seen in [6].

Let $P=\{D_0=1, D_1, \cdots, D_n\}$ be a finite subset of $End(B_A)$ and let P be a poset with respect to the order $\leq$. For D_i and D_j in P, $D_i \gg D_j$ means that D_i is a cover of D_j, that is, $D_i > D_j$ and no $D_k \in P$ such that $D_i > D_k > D_j$.

$P(min)$ (resp. $P(max)$) is the set of all minimal (resp. maximal) elements of P.

For $D_i \in P$, a chain of D_i means a descending chian in P such that
$D_i = D_{i_0} \gg \ldots\ldots \gg D_{i_m}$, $D_{i_m} \in P(min)$,
and $m+1$ is said to be the length of the chain.

(I) P is said to be a relative sequence of homomorphisms if it satisfies the following conditions (A. 1)-(A. 4) and (B. 1)-(B. 4):

(A. 1) $D_i \neq 0$ for all $D_i \in P$ and $P(min)$ coincides with all $D_i \in P$ such that D_i is a ring automorphism.

(A. 2) The length of each chain of D_i is unique and denotes it by $ht(D_i)$.

(A. 3) $D_iD_j \in P$ if $D_iD_j \neq 0$ and if $D_iD_j = 0$ then $D_jD_i = 0$.

(A. 4) Assume D_iD_j and D_iD_k are in P.

(i) $D_iD_j \geq D_iD_k$ (*resp.* $D_jD_i \geq D_kD_i$) if and only if $D_j \geq D_k$.

(ii) If $D_iD_j \geq D_m$ then $D_m = D_sD_t$ for some $D_s \leq D_i$ and $D_t \leq D_j$.