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Pointwise differentiability properties of solutions of quasilinear parabolic equations

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Abstract

We prove that weak solutions of the quasilinear equation

$$u_t - \mathrm{div}\mathscr{A}(t, x, u, u_x) = \mathscr{B}(t, x, u, u_x)$$

are at almost every point Lipschitz continuous in the space directions and Hölder continuous with exponent 1/2 in the time direction. This allows us to conclude that those weak solutions for which $u_t \in L^{\infty,1}_{loc}$ are totally differentiable almost everywhere. In the course of establishing these two theorems we generalize a result of Yu. G. Reshetnyak concerning the so -called $W^{m,p}$-differentiability of Sobolev functions

1. Introduction

In this paper, we study the pointwise differentiability (in the classical sense) of weak solutions of a class of quasilinear partial differential equations of parabolic type.

Let $X=(t, x_1, \ldots, x_n)$, $Y=(s, y_1, \ldots, y_n)$, $Z=(r, z_1, \ldots, z_n)$ denote points in $(n+1)$-dimensional Euclidean space $\boldsymbol{R}^{n+1}$. Let Ω be a bounded open domain in $\boldsymbol{R}^n$ and $G=(0, T)\times\Omega$. For $X \in G$, we consider the second order quasilinear equation

$$u_t - \mathrm{div}\mathscr{A}(t, x, u, u_x) = \mathscr{B}(t, x, u, u_x), \tag{1}$$

where $\mathscr{A}=(\mathscr{A}_1, \ldots, \mathscr{A}_n)$ is a $\boldsymbol{R}^n$-valued function of $(t, x, u, u_x) \in G\times\boldsymbol{R}\times\boldsymbol{R}^n$, $\mathscr{B}$ is a real-valued function of the same variables, $u_x=\left(\frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_n}\right)$ denotes the spatial gradient of the function u, and $\mathrm{div}\mathscr{A}$ stands for the divergence of the vector $\mathscr{A}(t, x, u(t, x), u_x(t, x))$ with respect to the space variables $x_1, \ldots, x_n$. Moreover, we assume that $\mathscr{A}(X, u(X), \psi(X))$ and $\mathscr{B}(X, u(X), \psi(X))$ are measurable for every choice of measurable func-

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