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The lifespan of classical solutions to nonlinear wave equations in two space dimensions

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§ 1. Introduction

In the present paper we study the lifespan of solutions to initial value problems for nonlinear wave equations of the form

$$(1.1)\qquad \begin{aligned}&\partial_t^2 u(x, t)-\Delta u(x, t)=A|u(x, t)|^p,\ (x, t)\in \boldsymbol{R}^n\times[0, \infty),\\ &u(x, 0)=f(x),\ \partial_t u(x, 0)=g(x),\ x\in \boldsymbol{R}^n,\end{aligned}$$

where p and A are positive constants and $n=2, 3$.

F. John [6] has proved the following remarkable results in three space dimensions. The global classical solution to (1.1) exists for small initial data with compact support provided $p>p_0(3)=1+\sqrt{2}$, and the lifespan of classical solution to (1.1) is finite provided $1<p<p_0(3)$, $f=0$, $g>0$ (also see F. John [7], p. 32). Here $p_0(n)$ stands for the positive root of the quadratic $q(n, p)=0$ where

$$(1.2)\qquad q(n, p)=\frac{n-1}{2}p^2-\frac{n+1}{2}p-1$$

and the lifespan of a solution u to (1.1) means the largest T such that $u(x, t)\in C^2(\boldsymbol{R}^n\times[0, T))$. He also proved in [6] that the lifespan T_ε of solutions to (1.1) with $f(x)=\varepsilon\varphi(x)$ and $g(x)=\varepsilon\psi(x)$ is equivalent to ε^{-2} for $p=2$. Recently H. Lindblad [9] has refined this result by showing that the following limit exists for $1<p<p_0(3)$:

$$\lim_{\varepsilon\to+0}\varepsilon^{-p(p-1)/q(3,p)}T_\varepsilon.$$

R. T. Glassey [3] has proved in two space dimensions that the global solution to (1.1) exists for small initial data with compact support provided $p>p_0(2)$. R. T. Glassey [4] proved that if $1<p<p_0(2)$ then the lifespan of a solution to (1, 1) is finite. Moreover, J. Schaeffer [10] proved that the lifespan is finite for critical values $p=p_0(2)$ and $p_0(3)$.

The main aim of this paper is to look for the upper and lower bounds