Hokkaido Mathematical Journal Vol. 21 (1992) p. 483-489

A variant of a Yamaguchi's result

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(Received November 5, 1991, Revised May 19, 1992)

Introduction

Yamaguchi extended the classical F. and M. Riesz theorem to a transformation group such that a compact abelian group acts on a locally compact Hausdorff space. In fact, he proved the following theorem.

THEOREM A. [5, Theorem 2.4] *Let (G, X) be a transformation group in which G is a compact abelian group and X is a locally compact Hausdorff space. Let σ be a positive Radon measure on X that is quasi-invariant and let Λ be a Riesz set in $\hat{G}$. Let μ be a measure in $\mathscr{M}(X)$ with spec $\mu\subset\Lambda$. Then spec μ_a and spec μ_s are both contained in spec μ, where $\mu=\mu_a+\mu_s$ is the Lebesgue decomposition of μ with respect to σ.*

Let us recall that a subset Λ of $\hat{G}$ is a Riesz set if $\mathscr{M}_\Lambda(G)\subset L^1(G)$ (where $\mathscr{M}_\Lambda(G)$ denotes the space of measures in $\mathscr{M}(G)$ whose Fourier transforms vanish off Λ). With this terminology, the classical F. and M. Riesz theorem asserts that $\boldsymbol{N}$ is a Riesz subset of $\boldsymbol{Z}$.

Godefroy introduced and studied the notion of nicely placed and Shapiro sets [2]. Let us recall the definitions.

DEFINITION 1. [2]

1. A subset Λ of $\hat{G}$ is said nicely placed if the unit ball of $L^1_\Lambda(G)$ is closed in measure.

2. A subset Λ of $\hat{G}$ is said Shapiro if every subset of Λ is nicely placed.

The Alexandrov's example shows that there exists a Riesz subset Λ of $\boldsymbol{Z}$ which is not nicely placed [2]: take $\Lambda=\bigcup_{n=0}^{\infty}D_n$, where $D_n=\{k2^n, |k|\leq 2^n, k\neq 0\}$. On the other hand of course $\boldsymbol{Z}$ is nicely placed in $\boldsymbol{Z}$ but not Riesz in $\boldsymbol{Z}$.

But Godefroy proved that every Shapiro set is a Riesz set [2].

Our aim is to show that the conclusion of theorem A also holds for another class of subsets Λ of $\hat{G}$: the nicely placed subsets. More precise-

[0]This work was supported by CGRI-FNRS-CNRS-grant.