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A note on the classification of nonsingular flows with transverse similarity structures

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§1. Introduction

The purpose of this paper is to classify nonsingular flows with transverse similarity structures satisfying certain auxiliary conditions. We consider such flows as foliations with transverse similarity structures and the classification is done in this view point. A motivation for this study is as follows. In Nishimori [7], the author investigated the qualitative properties of foliations with transverse similarity structures and gave an analogy of Sacksteder's theorem (in Sacksteder [8]) on codimension one foliations. Furthermore for such foliations, Matsuda [6] gave an analogy of a theorem of Hector and Duminy (in Hector [5] and in Cantwell and Conlon [1]) on codimension one foliations. So we are interested in concrete examples of foliations with transverse similarity structures.

Here we give the difinition of foliations with transverse similarity structures. A codimension q C^{∞} foliation $\mathscr{F}$ of a C^{∞} manifold M has a *transverse similarity structure* if there exsits an open covering $\{U_i\}_{i\in I}$ of M, a family $\{h_i: U_i \to \boldsymbol{R}^q\}_{i\in I}$ of C^{∞} submersions such that (1) $\mathscr{F}|_{U_i}=\{h_i^{-1}(t)\}_{t\in h_i(U_i)}$, and (2) for each $i,j\in I$ with $U_i\cap U_j\neq\emptyset$, there exists a similarity transformation $\gamma_{ji}: \boldsymbol{R}^q \to \boldsymbol{R}^q$ satisfying.

$$\gamma_{ji}\circ(h_i|_{U_i\cap U_j})=h_j|_{U_i\cap U_j}.$$

We call $\theta=\{U_i, h_i, \gamma_{ji}\}$ *a transverse similarity structure* of $\mathscr{F}$.

By starting from one of such submersions h_i's, we can construct the *analytic continuation* and obtain a C^{∞} submersion $D: \widetilde{M} \to \boldsymbol{R}^q$, where $\widetilde{M}$ is the universel covering of M. We call D a *developing map* of the foliation $\mathscr{F}$ with the similarity structure $\theta=\{U_i, h_i, \gamma_{ji}\}$. As is well known (see Godbillon [4] for example), there exists a homomorphism $\Phi: \pi_1(M) \to \mathrm{Sim}(q)$ such that $D\circ\gamma=\Phi(\gamma)\circ D$ for all $\gamma\in\pi_1(M)$, where $\mathrm{Sim}(q)$ is the group of similarity transformations of $\boldsymbol{R}^q$. In this paper, we work in the *oriented category* for simplicity. So we suppose that γ_{ji}'s are orientation preserving, that is, $\gamma_{ji}\in\mathrm{Sim}_+(q)$.

If the dimension of such $\mathscr{F}$ is zero, the triple $(M, \mathscr{F}, \{U_i, h_i, \gamma_{ji}\})$ is a