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An F. and M. Riesz theorem on locally compact transformation groups

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Dedicated to Professor TSUYOSHI ANDO on his sixtieth birthday

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§ 1. Introduction.

Helson and Lowdenslager extended the classical F. and M. Riesz theorem as follows.

THEOREM A (cf. [12, 8.2.3. Theorem]). *Let G be a compact abelian group with ordered dual, i. e., there exsits a semigroup P in $\widehat{G}$ such that (i) $P\cup(-P)=\widehat{G}$ and (ii) $P\cap(-P)=\{0\}$. Let μ be a measure in $M(G)$ such that $\hat{\mu}(\gamma)=0$ for $\gamma<0$. Then*

(I) $\widehat{\mu}_a(\gamma)=\widehat{\mu}_s(\gamma)=0$ *for* $\gamma<0$;

(II) $\widehat{\mu}_s(0)=0$.

Theorem A (I) was extended, by the author ([13]) and Hewitt-Koshi-Takahashi ([7]), to LCA groups as follows.

THEOREM B (cf. [13, Corollary], [7, Theorem D]).
Let G be a LCA group and P a semigroup in $\widehat{G}$ such that $P\cup(-P)=\widehat{G}$. Let μ be a measure in $M_p(G)$, where $M_p(G)=\{\nu\in M(G):\hat{\nu}=0$ on $P^c\}$. Then μ_a and μ_s also belong to $M_p(G)$.

In Theorem B we can not expect "$\widehat{\mu}_s(0)=0$" in general. As pointed out in the proof of [13, Corollary], Theorem B follows from the following theorem.

THEOREM C (cf. [13, Main Theorem]). *Let G be a LCA group and P a closed semigroup in $\widehat{G}$ such that $P\cup(-P)=\widehat{G}$. Let μ be a measure in $M_{p^c}(G)$. Then μ_a and μ_s also belong to $M_{p^c}(G)$.*

On the other hand, Forelli obtained the following theorem ([5]).

THEOREM D (cf. [5, Theorem 5]). *Let $(\boldsymbol{R}, X)$ be a (topological) transformation group, in which the reals $\boldsymbol{R}$ acts on a locally compact Hausdorff space X. Let σ be a positive Radon measure on X that is quasi-invariant. Let $\mu\in M(X)$, and let $\mu=\mu_a+\mu_s$ be the Lebesgue decomposi-*