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Extension of local direct product structures of normal complex spaces

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§ 0. Introduction

In [GM], Gómez-Mont defined foliations by curves on complex spaces. Let X be a complex space of pure dimension n and non-singular in codimension one, i. e. the singular locus $\mathrm{Sing}X$ is of codimension strictly greater than one. $X\backslash\mathrm{Sing}X$ is a (not necessarily connected) complex manifold of dimension n. We consider a pair $(\mathscr{F}_{A}, A)$, where A is an analytic set in X of codimension strictly greater than one and containing $\mathrm{Sing}X$ and $\mathscr{F}_{A}$ is a holomorphic foliation of complex dimension one on $X\backslash A$. (Note that $X\backslash A$ is an n-dimensional complex manifold.) Two pairs $(\mathscr{F}_{A}, A)$, $(\mathscr{F}_{A'}, A')$ are said to be equivalent if there exists an analytic set B of X of codimension strictly greater than one which contains $A\cup A'$, and if two foliations $\mathscr{F}_{A}|_{X\backslash B}$ and $\mathscr{F}_{A'}|_{X\backslash B}$ on $X\backslash B$ coincide with each other. A foliation $\mathscr{F}$ by curves on X is an equivalence class of such a pair $(\mathscr{F}_{A}, A)$. Note that if X is normal and connected then X is pure dimensional and non-singular in codimension one.

After Gómez-Mont's definition, we may say that the "simplest" foliation by curves on X is defined by a pair $(\mathscr{F}_{A}, A)$ such that $\mathscr{F}_{A}$ is a direct product, i. e. for an $(n-1)$-dimensional complex manifold M and a Riemann surface S,

$$X\backslash A\simeq M\times S$$

as complex manifolds.

In this parer, we confine our interest to the local case and investigate the following problem:

PROBLEM 0. 0.
Let X be a complex space and $x_{0}\in X$. Suppose that there are an open neighborhood $U\subset X$ of x_{0}, an analytic set A in U, a complex space V and a Riemann surface W satisfying the following conditions:

a) $x_{0}\in A$,

b) $\mathrm{codim}A>1$ (in U) and