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# Martin boundaries and thin sets for $\Delta u = Pu$ on Riemann surfaces

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## 1. Introduction.

Let $R$ be a hyperbolic Riemann surface and $P$ a density on $R$, that is, a non-negative Hölder continuous function on $R$ which depends on the local parameter $z=x+iy$ in such a way that the partial differential equation

$$L_P u \equiv -\Delta u + Pu = 0, \quad \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 \tag{0.1}$$

is invariantly defined on $R$. A real valued function $u$ is said to be a $P$-harmonic function (or a $P$-solution) on an open set $U$ of $R$, if $u$ has continuons partial derivatives up to the order 2 and satisfies the equation (0.1) on $U$. Throughout this paper we assume that the density $P$ is not constantly zero on $R$. A density $P$ on a Riemann surface $R$ is called a hyperbolic density if there exists a positive $P$-harmonic function on $R$ dominated by 1 on $R$. In this paper we shall consider two Martin compactifications of a hyperbolic Riemann surface $R$, the first $R_P^*$ with respect to a hyperbolic density $P$ on $R$, the second $R^*$ with respect to harmonic functions. Let $K^P(z,a)$, $K(z,b)$ be Martin kernels on the compactifications $R_P^*$, $R^*$ respectively. Let $G(z,w)$ be the harmonic Green's function of $R$. For a minimal boundary point $a$ of $R_P^*$ such that

$$\int_R P(w)G(w,z_1)K^P(w,a)dudv < +\infty \tag{0.2}$$

for some point $z_1$ in $R$, there exists a unique minimal boundary point of $R^*$. Then we may define a mapping with the domain consisting points $a$ which satisfy the condition (0.2) into the set of minimal boundary points of $R^*$.

In the special case where $P$ is a bounded rotation free density on the unit disk in complex plane $C$, two compactifications coincide with the closed unit disk $\{z \in C : |z| \leq 1\}$. In this case our mapping reduces to the identity mapping of the unit circle (Remark in sec. 2). The purpose of this paper is to show that a closed set $E$ in $R$ is thin at a poinnt $a$ with