Blow-up for some equations with semilinear dynamical boundary conditions of parabolic and hyperbolic type

M. KIRANE
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1. Introduction

In recent years several results have been published concerning blow-up of solutions to semilinear parabolic and hyperbolic equations. We mention here the works [1]-[7] and their references.

A special attention was given not only to the description of the set of blow-up points but also to the description of the behaviour of the solutions near the blow-up points as time tends to the blow-up time [3]-[6].

All these papers dealt with semilinear parabolic and hyperbolic equations with classical boundary conditions, i. e, Dirichlet's, Neumann's and Robin's conditions.

In the present work blow-up results and characterization of the blow-up set (Bus) in the case of particular geometries are established for semilinear parabolic and hyperbolic equations of the following types:

(P1)
$$\triangle u = 0 \qquad \text{in } D \times (0, \infty),$$
$$\partial u / \partial t + k \partial u / \partial \eta = h(x, t, u) \qquad \text{on } S \times (0, \infty),$$
$$u(x, 0) = u_0(x) \qquad \text{on } S.$$

(P2)
$$\triangle u = 0 \qquad \text{in } D \times (0, \infty),$$
$$\partial^2 u / \partial t^2 + k \partial u / \partial \eta = f(u) \qquad \text{on } S \times (0, \infty),$$
$$u(x, 0) = u_0(x) \qquad \text{on } S,$$
$$\partial u / \partial t(x, 0) = u_1(x) \qquad \text{on } S.$$

(P3)
$$\partial u / \partial t - \triangle u = u^{1+\alpha} \qquad \text{in } D \times (0, \infty),$$
$$\partial u / \partial t + k \partial u / \partial \eta = u^{1+\alpha} \qquad \text{on } S \times (0, \infty),$$
$$u(x, 0) = u_0(x) \qquad \text{in } \bar{D}.$$

Here D is a bounded domain in R^N ($N \geq 1$) with smooth boundary S and outer unit normal vector field η, $\triangle$ is the Laplace operator with respect to the space variables and $\partial / \partial \eta$ the outward normal derivative to S. The constants α and k are assumed to be positive.

The functions h and f are assumed to satisfy: