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A qualitative theory of similarity pseudogroups and an analogy of Sacksteder's theorem

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1. Introducion

The qualitative theory of foliations has been developed for foliations of codimension one (see Sacksteder [7], Cantwell-Conlon [1] and Hector [4] for example). Now we intend to study qualitative properties of foliations of higher codimensions. Note that all the non-singular dynamical systems can be considered as foliations and there are numberless researches on the qualitative theory of dynamical systems. Such researches are not our intention. So we must make our purpose more concrete. The most typical result in the qualitative theory of codimension one foliations is the following theorem.

THEOREM (Sacksteder's Theorem, see Sacksteder [7]). *Let $\mathcal{F}$ be a codimension one C^2 foliation of a closed manifold M, and $\mathcal{M} \subset M$ an exceptional minimal set with respect to $\mathcal{F}$. Then there exists a leaf F of $\mathcal{F}$ contained in $\mathcal{M}$ such that F has a contracting element in its linear holonomy group* $\mathrm{LHol}(F)$.

We demand that our intended study should contain an analogy of the above theorem, and look for an appropriate and simple category of foliations on which we should work. A natural idea is to consider foliations with transverse geometric structure (see Godbillon [3] for example). The automorphism groups of the appropriate geometric structures are requested to contain contracting elements for an expected analogy of Sacksteder's theorem. These considerations guide us to investigate foliations with transverse similarity structure (see Ghys [2] and Nishimori [6]).

In this paper, we are going to treat similarity pseudogroups Γ on $\boldsymbol{R}^q$ in place of codimension q foliations $\mathcal{F}$ with tranverse similarity structure. As is well known, there exist natural correspondences between the terms in the qualitative theories of these objects. For example, one considers Γ-orbits in place of leaves of $\mathcal{F}$, and the stabilizer at a point in a Γ-orbit