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Standard spatial graph

Dedicated to Professor Haruo Suzuki on his 60th birthday

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§ 0. Introduction

In this article we consider the graph as a topological space or a CW complex and define some kinds of standard embeddings of the graphs into the 3-dim. Euclidean space R^3 or the 3-sphere S^3 and discuss the property. For example, for a knot or a link we can define the standard embedding the trivial knot or the trivial link. And although, by Fox's Theorem, for any finite graph G there is a spatial graph $\tilde{G}$ of G such that the complementary space of the interior of a regular neighborhood of $\tilde{G}$ is homeomorphic to a handlebody with genus equal to the rank $H_1(G:Z)$, this spatial graph is not suitable to the "*standard*" spatial graph in general (see Figure 6). Furthermore the image of any embedding of the complete graph with n vertices, $\tilde{K}_n$, contains a non-trivial link for $n \geqq 6$ and contains a non-trivial knot for $n \geqq 7$ ([2]). So if we adopt as the definition of the standard embedding a spatial graph (i. e. the embedded image of the graph) which does not contain a non-trivial knot or link, the definition can not apply to all finite graph. Although there are concepts of minimal genus, maximal genus and thickness in the graph theory ([1]), [11]), these concepts do not satisfy the properties which should be satisfied by the "*standard*" embedding from view point of the knot theory. That is, the graph theoretical properties of the above are weak for the "*standard*" embedding from the knot theory. For example, although two spatial graphs of the complete graph K_5 of Figure 4 and 5 are both on the torus but the first one does not contain any non-trivial knot and the second one contains a trefoil knot.

From the knot theory, the properties which should be satisfied by the "*standard embedding*" (or the "*standard spatial graph*") are the following;

Let G be a finite graph and $\tilde{G}$ be a "*standard*" spatial graph of G. Then