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Higher dimensional semilinear parabolic problems

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1. Introduction

There are two commonly used ways of approaching parabolic problems, i.e. "dynamic" semigroup technique (e.g. [AM], [WA]) and "static" a priori estimates method (e.g. [LA], DL]). In spite of the great power of "dynamical" approach (which leads to the general results for the possibly wide class of nonlinear problems), classical in the theory of partial differential equations "static" a priori technique can very often give exact in form and precise in assumptions existence-uniqueness theorems concerning regular solutions in the space of Hölder functions. Moreover dealing with the problem of local solvability one is also able to estimate (from below) the "life time" of the obtained solution in a way similar to the well known Peano theorem in the theory of ordinary differential equations.

However both in the "dynamic" and "static" studies of the classical solvability, growth of the space dimension n causes a feedback in the sense of increase (with respect to n) of the assumptions that should be put on the data of the considered problem. Since referring to needed assumptions as "sufficiently regular" makes the final result hardly applicable, we want in this note to deal with the case of higher space dimension coming back to the idea of our recent paper [CH], in which higher dimensional case was only mentioned in the Appendix.

We have presented in [CH] a classical approach to the $2m$-th ($m>1$) order initial-boundary value problem

$$(1)\qquad \begin{cases} u_t=-Pu+f(t,x,d^m u) & \text{in } D^T=(0,T)\times G \\ B_0u=\ldots=B_{m-1}u=0 & \text{on } \partial G \\ u(0,x)=u_0(x) & \text{in } G \end{cases}$$

with $P=\sum_{|\alpha|,|\beta|\le m}(-1)^{|\beta|}D^\beta(a_{\alpha,\beta}(x)D^\alpha)$, $d^m u=\left(u, \frac{\partial u}{\partial x_1}, \cdots, \frac{\partial u}{\partial x_n}, \frac{\partial^2 u}{\partial x_1^2}, \cdots, \frac{\partial^m u}{\partial x_n^m}\right)$ and a bounded domain $G\subset R^n$ having smooth $C^{4m+\mu}$ boundary ∂G, where $\mu\in(0,1)$ is fixed from now on.

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