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Finite time blowing-up for the Yang-Mills gradient flow in higher dimensions

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1. Introduction.

Let $n\geq 5$, and P be a non-trivial principal G-bundle over S^n with the standard metric g, where G is a compact Lie group satisfying $G\subset SO(N)$. In this paper, we prove the solution of the evolution problem for Yang-Mills connections may blow up in finite time. Yang-Mills connections over P are critical points of the functional

$$E(D)=\frac{1}{2}\int_M |F(D)|^2 dV,$$

where $F(D)$ is the curvature form of connection D. If D is a Yang-Mills connection, then it satisfies the Euler-Lagrange equation of E:

$$d_D^* F(D)=0,$$

where d_D^* is the formal adjoint operator of the exterior derivative d_D with respect to the connection D.

In this paper, we consider the Yang-Mills gradient flow:

$$(1.1)\qquad \begin{cases} \dfrac{\partial D}{\partial t}=-d_D^* F(D), & \text{on } M\times[0, T) \\ D(0)=D_0. \end{cases}$$

In the fundamental work of Donaldson [7], he showed the global existence of the heat flow on a holomorphic vector bundle over a compact Kähler manifold. Recently, Kozono, Maeda and the author [8] show the existence of a global weak solution for the heat flow, if dim $M=4$. It is a well-known result that if the initial value D_0 is smooth, then there exists a time-local smooth solution $D(x, t)$ of (1.1) on $M\times[0, T)$ for any compact Riemannian manifold M with arbitrary dimension. In higher dimensional case (dim $M\geq 5$), Bourguignon, Lawson and Simons [2] and Bourguignon and Lawson [1] showed isolation phenomena of Yang-Mills connections over S^n.

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