

## Note on purifiable subgroups of primary abelian groups

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**Abstract.** Let  $A$  be a purifiable subgroup of an abelian  $p$ -group  $G$  and  $H$  be a pure hull of  $A$  in  $G$ . Then  $H$  is a direct summand of  $G$  if and only if  $G[p]/A[p]$  is purifiable in  $G/A[p]$ . In addition, if  $H$  is a direct summand of  $G$ , then all pure hulls of  $A$  are direct summands of  $G$ , there exists the same complementary summand of  $G$  for every pure hull of  $A$ , and all pure hulls of  $A$  are isomorphic.

*Key words:* purifiable subgroup, pure hull, direct summand, vertical subgroup,  $m$ -vertical subgroup.

All groups considered here are  $p$ -primary abelian groups for a fixed prime number  $p$ . Throughout this note, let  $A$  be a subgroup of a group  $G$ .

$A$  is said to be purifiable in  $G$  if there exists a pure subgroup  $H$  of  $G$  containing  $A$  which is minimal among the pure subgroups of  $G$  that contain  $A$ . Such a subgroup  $H$  is said to be a pure hull of  $A$  in  $G$ . In a direct sum of cyclic groups, every subsocle is purifiable.

Let  $S$  be a subsocle of  $G$ . J. Irwin and J. Swanek have shown in [6] that if  $G/S$  is a direct sum of cyclic groups and  $S$  supports a pure subgroup  $H$ , then  $G$  is a direct sum of cyclic groups and  $H$  is a direct summand of  $G$ . Furthermore, they also have characterized pure subgroups to be direct summands of a given group in [6].

In Section 2, we consider their problems on the assumptions which extend subsocles to purifiable subgroups and pure subgroups to purifiable subgroups in a given group. Then we obtain that a pure hull of a purifiable subgroup is a direct summand of a given group  $G$ , but  $G$  is not necessarily a direct sum of cyclic groups. We give such an example. Moreover, we characterize a purifiable subgroup  $A$  of  $G$  that a pure hull of  $A$  is a summand of  $G$ . Using this result, we generalize several results of J. Irwin and J. Swanek's.

It is well-known that all pure hulls of a subsocle in a direct sum of