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Sheaf cohomology theory for measurable spaces

Hideyuki KIKUCHI
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1. Introduction

Sheaf theory was mainly applied to topology, differential geometry, algebraic geometry, and so on (see G. E. BREDON [1], R. G. SWAN [2] and J. DIEUDONNÉ [7]). For example in topology, it is very useful in proving theorems such as the duality theorems of POINCARÉ, ALEXANDER, and LEFSCHETZ. It is important when we want to obtain global properties from local ones. It takes more effect when combined with homological algebra, especially cohomology theory. For instance, cohomology theory is used in defining characteristic classes and cohomology vanishing theorem is useful in calculating them.

The aim in this paper is to develop a sheaf cohomology theory for a measurable space $(\Omega, \mathfrak{A})$. To each σ-subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ we associate an abelian group $\mathscr{F}(\mathfrak{B})$ and call the system of them *σ-sheaf* $\mathscr{F}$ over $(\Omega, \mathfrak{A})$. We formulate cohomology group with coefficients in it. We treat mainly a cohomology group with coefficients in a σ-sheaf of measurable transformation group or automorphism on $(\Omega, \mathfrak{A})$. It gives certain relation between the local characteristics and the global ones of transformation group on $(\Omega, \mathfrak{A})$. We show cohomology vanishing theorems with respect to it.

We state a summary of each section below.

In Section 2, we give the definition of σ-sheaf which plays a role of describing the local-global interplay. We construct two kinds of σ-sheaves: one is a σ-sheaf of measurable transformation group and the other is a σ-sheaf of integrable functions over a finite measure space.

Section 3 constructs a cohomology group with coefficients in a σ-sheaf $\mathscr{F}$ in the similar way to the construction of the Čech cohomology for topological space. To irrustlate cohomology group, we regard the σ-algebra $\mathfrak{A}$ and σ-subalgebras of $\mathfrak{A}$ as the domain and its sub-domains, respectively. By a *σ-covering* over $(\Omega, \mathfrak{A})$, we mean a collection of σ-subalge-

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