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A note on duality of first order partial differential equations

Kenji AOKI

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0. Introduction

The dual relationships among the equations are given by the Legendre transformation in the classical theory of differential equations and it is useful to solve some type of differential equations [**5**]. However situations are not so clear in the classical theory as usual. So Izumiya establishes the principle of duality among first order ordinary differential equations with complete solutions [**6**]. Our purpose in this note is to generalize the result for systems of first order partial differential equations with complete solutions.

The geometrical theory of first order partial differential equations is described natually in the context of contact geometry, which can be considered as a generalization of projective geometry ([**1**], [**2**]). A particular aspect of projective geometry is the principle of duality. So we may expect that some type of duality holds also among first order partial differential equations.

In § 1 we shall prepare some basic notions and construct the framework. In § 2 we shall establish the principle of duality among pairs of completely integrable system of first order partial differential equations and its complete solution. In the special case of holonomic systems of first order partial differential equations, we can assert a more strong result (i.e. the principle of duality among completely integrable holonomic systems of first order partial differential equations themselves), which is discussed in § 3.

All arguments should be understood locally and all maps considered here are differentiable of class C^∞.

1. Basic notions

In this section we shall state our basic notions. *A system of partial differential equations of first order* (or briefly, *an equation*) is a submersion germ $F:(J^1(\boldsymbol{R}^n, \boldsymbol{R}), z_0)\to(\boldsymbol{R}^d, 0)$, $1\leq d\leq n$, on the 1-jet space of functions of n-variables. If $d=n$, we call it *an holonomic system of par-*