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Interpolating sequences and embedding theorems in weighted Bergman spaces

Masahiro YAMADA

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Abstract. For $0<p<\infty$, let $L^{p}{}_{a}(\mu)$ denote the weighted Bergman space on the unit disk D in the complex plane, where μ is a finite positive Borel measure on D. When μ is an absolutely continuous measure which satisfies an (A_{p})-condition, we study interpolating sequences on $L^{p}{}_{a}(\mu)$ and give several sufficient conditions in order that such a sequence exists in $L^{p}{}_{a}(\mu)$. Using them, we obtain embedding theorems for weighted Bergman spaces between $L^{p}{}_{a}(\mu)$ and $L^{q}{}_{a}(\nu)$, where ν is a finite positive Borel measure on D and $0<q<\infty$.

Key words: interpolating sequence, (A_{p})-condition, Carleson inequality, Bergman space, analytic function.

1. Introduction

Let D denote the open unit disk in the complex plane and H a set of all analytic functions on D. For $0<p<\infty$, let $L^{p}(\mu)$ denote an L^{p}-space on D with respect to a finite positive Borel measure μ on D and set $L^{p}{}_{a}(\mu)=L^{p}(\mu)\cap H$, which is called a weighted Bergman space on D.

For any a in D, let ϕ_{a} be the Möbius function on D, that is, $\phi_{a}(z)=(a-z)/(1-\bar{a}z)$ $(z\in D)$, and put $\beta(a,z)=1/2\{\log(1+|\phi_{a}(z)|)(1-|\phi_{a}(z)|)^{-1}\}$ $(a,z\in D)$. For $0<r<\infty$ and a in D, let $D_{r}(a)=\{z\in D;\beta(a,z)<r\}$ be the Bergman disk with "center" a and "radius" r, and m be the Lebesgue area measure on D. We define an average of a finite positive measure μ on $D_{r}(a)$ by

$$\widehat{\mu}_{r}(a)=\frac{1}{m(D_{r}(a))}\int_{D_{r}(a)}d\mu \quad (a\in D),$$

and if there exists a non-negative function w in $L^{1}(m)$ such that $d\mu=wdm$, then we may write it $\widehat{w}_{r}$ instead of $\widehat{\mu}_{r}$.

Let ν and μ be finite positive Borel measures on D, and for $0<p$, $q<\infty$, let $i:L^{p}{}_{a}(\mu)\to L^{q}{}_{a}(\nu)$ be an inclusion mapping. Our purpose of this paper is to study a necessary and sufficient condition on ν and μ so

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