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Rigidity theorems for real hypersurfaces in a complex projective space

(Dedicated to Professor Tsunero Takahashi on his 60th birthday)

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Abstract. We prove two rigidity theorems for real hypersurfaces in $P_n(\mathbb{C})$. More precisely, let M be a $(2n-1)$-dimensional Riemannian manifolds, and ι and $\hat{\iota}$ be two isometric immersions of M into $P_n(\mathbb{C})$. Then ι and $\hat{\iota}$ are congruent if the type number of ι and $\hat{\iota}$ is not equal to 2 everywhere, and moreover (a) two structure vector fields coincide up to sign or (b) there exists an m-dimensional subspace of the tangent space of M at each point invariant under the actions of the two shape operators of ι and $\hat{\iota}$ $(2 \leq m \leq n-1)$.

Key words: rigidity, structure vector, shape operator.

Introduction

Let $P_n(\mathbb{C})$ be an n-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature $4c$ and M be a $(2n-1)$-dimensional Riemannian manifold. Let ι be an isometric immersion of M into $P_n(\mathbb{C})$. An *almost contact structure* on M induced from the complex strcture $\tilde{J}$ of $P_n(\mathbb{C})$ by ι will be denoted by (ϕ, ξ) and ξ is called the *structure vector field* of ι.

The last named author proved in [5] that two isometric immersions of M into $P_n(\mathbb{C})$ are rigid if their second fundamental forms coincide. Recently, the same author and Y.J. Suh [4] also obtained the same conclusion if the two isometric immersions have a principal direction in common and type number is not equal to 2 at each point of M, where the *type number* is defined as the rank of the second fundamental form.

In this paper we shall study some conditions for two isometric immersions of M into $P_n(\mathbb{C})$ to be rigid. The main purpose is to prove the following

Theorem A *Let M be a $(2n-1)$-dimensional Riemannian manifold, and ι and $\hat{\iota}$ be two isometric immersions of M into $P_n(\mathbb{C})$ $(n \geq 3)$. If the two structure vector fields coincide up to sign on M and the type number*

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