Hokkaido Mathematical Journal Vol. 25 (1996) p. 387–405

On a certain property of closed hypersurfaces with constant mean curvature in a Riemannian manifold, II

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(Received September 8, 1995)

Abstract. In this paper, we discuss some properties of a closed hypersurface whose first mean curvature is constant, in a Riemannian manifold admitting a special concircular scalar field.

Key words: hypersurface with constant mean curvature, special concircular scalar field, umbilic point, beging isometric to a sphere.

1. Introduction

Y. Katsurada [2] proved.

Theorem 1.1 (Katsurada) *Let R^{n+1} be an $(n+1)$-dimensional Einstein manifold which admits a proper conformal Killing vector field ξ^i, that is, a vector field generating a local one-parameter group of conformal transformations, and V^n a closed orientable hypersurface in R^{n+1} such that*
(i) *its first mean curvature H_1 is constant,*
(ii) *the inner product $C^i\xi_i$ has fixed sign on V^n,*
where C^i and ξ_i denote the normal vector to V^n and the covariant components of the conformal Killing vector field ξ respectively. Then every point of V^n is umbilic.

To prove Theorem 1.1, we need integral formulas of Minkowski type for a hypersurface in a Riemannian manifold in which the conformal Killing vector field plays the same role as the position vector in a Euclidean space.

We can prove that if every point of a closed orientable hypersurface in a Euclidean space is umbilic, then the hypersurface is isometric to a sphere. However, in a Riemannian manifold, we can not expect the result of the same kind even if every point of a closed orientable hypersurface is umbilic. On this problem, she [3] also proved the following two Theorems:

1991 Mathematics Subject Classification : 53C40.