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Positive values of inhomogeneous indefinite ternary quadratic forms of type (2,1)

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Abstract. Let $\Gamma_{2,1}^{(k)}$ denote the k^{th} successive inhomogeneous minima for positive values of real indefinite ternary quadratic forms of type (2,1). Earlier the first four minima for the class of zero forms were obtained. Here it is proved that for all the forms, whether zero or non zero, $\Gamma_{2,1}^{(2)}=8/3$. All the critical forms have also been obtained.

Key words: inhomogeneous minimum, quadratic forms, lattices, admissible, continued fractions.

1. Introduction

Let $Q(x_1, x_2, \ldots, x_n)$ be a real indefinite quadratic form in n variables of determinant $D \neq 0$ and of type $(r, n-r)$. Let $\Gamma_{r,n-r}$ denote the infimum of all numbers $\Gamma > 0$ such that for any real numbers $c_1, c_2, \ldots, c_n$ there exist integers $x_1, x_2, \ldots, x_n$ satisfying

$$0 < Q(x_1 + c_1, x_2 + c_2, \ldots, x_n + c_n) \leq (\Gamma |D|)^{1/n}. \tag{1.1}$$

The values of $\Gamma_{r,n-r}$ are known for various n. See for reference Aggarwal and Gupta [1]. Let $\Gamma_{r,n-r}^{(k)}$ denote the k^{th} successive inhomogeneous minimum for positive values of indefinite quadratic forms of type $(r, n-r)$. Bambah et al [2] proved that $\Gamma_{2,3}^{(2)} = 16$. Dumir and Sehmi [5, 6] obtained $\Gamma_{r+1,r}^{(2)}$ for all $r \geq 2$. For incommensurable forms (forms that are not multiple of rational forms) (1.1) is true with arbitrary small constant by a result of Watson [13] and Oppenheim's conjecture proved by Margulis [10]. Rational forms in $n \geq 5$ variables are zero forms by Meyer's Theorem. Ternary and quaternary forms are not necessarily zero forms. So Dumir and Sehmi [5, 6] just needed to consider zero forms. Raka [9] obtained the first four minima for ternary forms of the type $(2, 1)$ for the class of zero forms. For zero forms there is a standard method using a result of Macbeath [8].

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