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A note on the decomposition of the Burnside rings of finite groups

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Abstract. The Burnside ring $\Omega(G)$ of a finite group G has, as an abelian group, a decomposition $\Omega(G)=\Omega(G,\mathcal{X})\oplus K(\mathcal{X})$ where $K(\mathcal{X})$ is an ideal and $\Omega(G,\mathcal{X})$ is the generalized Burnside ring with respect to a family $\mathcal{X}$ of subgroups of G.

Key words: Burnside ring, p-locally determined function, Alperin's conjecture.

1. Introduction

In 7.2 of [Th], J. Thévenaz showed the following theorem for the Burnside ring $\Omega(G)$ of a finite group G.

Theorem (7.2 [Th]) *Let $\mathcal{X}$ be the family of subgroup H of G such that $O_p(H)$ is not trivial. Then*

$$\Omega(G)=B_{\mathcal{X}}\oplus K_{\mathcal{X}},$$

where

$$B_{\mathcal{X}} = \Big\{\sum_{S\in\mathcal{X}}\lambda_S[G/S]\mid\lambda_S\in\boldsymbol{Z}\Big\},$$
$$K_{\mathcal{X}} = \bigcap_{S\in\mathcal{X}}\mathrm{Ker}(\varphi_S).$$

Here $\varphi_S:\Omega(G)\rightarrow\boldsymbol{Z}$ is defined for each G-set X by $\varphi_S(X)=|X^S|$, the number of S-fixed points in X. The above is equivalent to 3.1 of [Th], which is a key result for showing that Alperin's conjecture [Al] and the following assertion on p-locally determined functions are equivalent.

Conjecture (Thévenaz [Th]) *If we write*

$$k(H)-z(H)=\frac{1}{|H|}\sum_{S\leq H}f_1(S)$$

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