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Remark on fundamental solution for vorticity equation of two dimensional Navier – Stokes flows

(Dedicated to Professor Kôji Kubota on his sixtieth birthday)

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Abstract. In this paper we treat a perturbed heat equation related to the vorticity equation for the Navier–Stokes flow in $\boldsymbol{R}^2$. We get estimate for the fundamental solution of this equation. We note that estimate like ours played the essential role in the paper by Giga, Miyakawa and Osada [4] where they discussed existence of solution for Navier–Stokes equation in $\boldsymbol{R}^2$ with measure as initial vorticity.

Key words: the incompressible Navier–Stokes equations, vorticity equation, fundamental solution, 2 dimensional flow.

1. Introduction and Results

Consider the incompressible Navier–Stokes equations in two dimensional Euclidean space $\boldsymbol{R}^2$:

$$\text{(NS)} \quad \begin{cases} u_t - \nu\Delta u + (u, \nabla)u + \nabla p = 0, \quad \operatorname{div} u = 0 & \text{in } (0,\infty) \times \boldsymbol{R}^2, \\ u|_{t=0} = u_0 \quad \text{in } \boldsymbol{R}^2, \end{cases}$$

where $u = u(t,x) = (u_1(t,x), u_2(t,x))$ is the velocity vector field, $p = p(t,x)$ is the pressure, $\nu > 0$ is the kinematic viscosity, $u_t = \partial u/\partial t$, $\nabla = (\partial/\partial x_1, \partial/\partial x_2)$ and $\operatorname{div} u = \partial u_1/\partial x_1 + \partial u_2/\partial x_2$.

For the vorticity $\omega(t,x) = \operatorname{rot} u(t,x) = \partial u_1/\partial x_2 - \partial u_2/\partial x_1$, we reduce (NS) to the following equations by the well known Biot–Savart law:

$$\text{(NSR)} \quad \begin{cases} \omega_t - \nu\Delta\omega + (u,\nabla)\omega = 0, \quad u(t,x) = \mathbf{K} * \omega(t,x) \\ \qquad\qquad\qquad\qquad\qquad \text{in } (0,\infty) \times \boldsymbol{R}^2, \\ \omega|_{t=0} = \omega_0 \equiv \operatorname{rot} u_0 \qquad \text{in } \boldsymbol{R}^2, \end{cases}$$

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