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Stability of symmetric systems under hyperbolic perturbations

(Dedicated to Professor Rentaro Agemi on his sixtieth birthday)

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Abstract. Let $L(x)$ be the symbol of a $m \times m$ symmetric first order hyperbolic system with real constant coefficients. The range of $L(x)$ is a subspace, containing a positive definite $L(\theta)$, in the linear space of dimension $d(m) = m(m+1)/2$ of all $m \times m$ real symmetric matrices. We study a hyperbolic perturbation $\tilde{L}(x) = L(x) + R(x)$ of $L(x)$, that is $R(x)$ is $O(|x|^2)$ $(x \to 0)$ which is real analytic and all eigenvalues λ of $\tilde{L}(x+\lambda\theta)$ are real near the origin. We prove that if the dimension of the range of $L(x)$ is greater than $d(m)-m+2$, then generically, every such hyperbolic perturbation is trivial, namely there are real analytic $A(x)$, $B(x)$ near the origin with $A(0)B(0) = I$ such that $A(x)\tilde{L}(x)B(x)$ becomes symmetric. When $m = 3$, the same conclusion holds if the range is greater than 3.

Key words: hyperbolic perturbation, symmetric system, non-degenerate.

1. Introduction

Let

$$\mathcal{L}(x) = \sum_{j=1}^{n} A_j x_j, \quad x = (x_1, \dots, x_n),$$

where A_j are real symmetric $m \times m$ matrices which are linearly independent. Since we are interested in hyperbolic systems we assume that $\mathcal{L}(\Theta)$ is positive definite with some $\Theta \in \mathbf{R}^n$. We may suppose that $\mathcal{L}(\Theta) = I$ considering $\mathcal{L}(\Theta)^{-1/2}\mathcal{L}(x)\mathcal{L}(\Theta)^{-1/2}$. The range $\mathcal{L} = \{\mathcal{L}(x) \mid x \in \mathbf{R}^n\}$ of $\mathcal{L}(x)$ is a linear subspace in $M^s(m, \mathbf{R})$, the space of all real symmetric $m \times m$ matrices. Note that the range contains the identity I and of n dimensional because A_j are linearly independent.

We study the symbol $\mathcal{P}(x)$ of a hyperbolic system which is *close* to $\mathcal{L}(x)$ near $x = 0$;

$$\mathcal{P}(x) = \mathcal{L}(x) + R(x)$$

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