Hokkaido Mathematical Journal Vol. 26 (1997) p. 203–209

A remark on the action of $PGL(2,q)$ and $PSL(2,q)$ on the projective line

(Dedicated to Professor Takeshi Kondo on his sixtieth birthday)

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(Received December 25, 1995)

Abstract. Let q be a prime power, $K=GF(q)$ the finite field with q elements, $\Omega=K\cup\{\infty\}$ the project line over K. Let 大 $=PGL(2,q)$ and 小 $=PSL(2,q)$ be the linear fractional group on Ω and the special linear fractional group on Ω, respectively. Let U be any non-trivial subgroup of the (cyclic) multiplicative group $K\backslash\{0\}$ and set $E=U\cup\{\infty\}$. The main purpose of this note is to determine the structures of 大$_E$ and 小$_E$, the setwise stabilizer of E in 大 and 小, respectively. Then, as an application, by taking various q and U, we obtain various 3-designs (Ω, $E^{大}$) and 3 (resp. 2)-designs (Ω, $E^{小}$) in case $q\equiv-1$, (resp. $q\equiv 1$) (mod 4), which contain new designs.

Key words: $PGL(2,q)$, $PSL(2,q)$, stabilizer, Frobenius group, design.

1. Introduction and notation

Throughout this note, we fix the following notation.

p :	any prime number
q :	a power of p
$K:=GF(q)$	finite field with q elements
$\Omega:=K\cup\{\infty\}$	projective line over K
$F:=K\backslash\{0\}$	multiplicative group of K
大[1] $:=PGL(2,q)=$	$\{x\mapsto(ax+b)/(cx+d)\mid a,b,c,d\in K,$ $ad-bc\in F\}$
小[2] $:=PSL(2,q)=$	$\{x\mapsto(ax+b)/(cx+d)\mid a,b,c,d\in K,$ $ad-bc\in F^2\}$
m :	a divisor of $q-1$ with $m>1$
U :	a subgroup of order m of the (cyclic) group F
$E:=U\cup\{\infty\}$	

1991 Mathematics Subject Classification : 20E07, 05B05.

1) '大' (dai) means 'large'.

2) '小' (shou) means 'small'.