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Automorphisms and conjugate connections

Mitsuhiro ITOH

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Abstract. We study how the automorphism group of a Lie group G acts on the space of gauge-equivalence classes of connections on a principal G-bundle P provided P is reducible to H-subbundles. The action is investigated in terms of conjugate connections and holonomy groups.

Key words: conjugate connection, automorphism group, holonomy group.

1. Introduction

Let P be a principal bundle over a manifold M with a structure group G.

We consider the situation that the structure group G of the bundle P happens to be reduced to a closed subgroup H.

Let σ be an automorphism of G leaving fixed all elements of H.

It turns out then that σ induces via the subgroup reduction a transformation of the space of connections on P, called a conjugation, a generalization of the notion of conjugate affine connection in affine differential geometry ([7]). The notion of conjugate connection in a principal bundle is due to S. Kobayashi and E. Shinozaki ([5]).

It is a principal question how this conjugation relates to the gauge theory of connections on a principal bundle.

S. Kobayashi and Shinozaki gave in [5] a substantial answer to this question as follows.

Theorem ([5]) *Let* $\mathrm{Aut}(G, H)$ *be the group of automorphisms of* G *leaving fixed each element of* H. *Let* $\mathcal{G}(P)$ *be the group of gauge transformations of* P, *and let* $\mathcal{C}(P)$ *be the space of connections on* P. *Then*

(i) *the group* $\mathrm{Aut}(G, H)$ *acts on* P, $\mathcal{G}(P)$ *and* $\mathcal{C}(P)$ *in a natural way so that it induces an action on* $\mathcal{C}(P)/\mathcal{G}(P)$,

and

(ii) *for a fixed Riemannian metric on an oriented* M *and for an au-*

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