Hokkaido Mathematical Journal Vol. 26 (1997) p. 63–88

Axisymmetric solutions and singular parabolic equations in the theory of viscosity solutions

(Dedicated to Professor Kôji Kubota on his sixtieth birthday)

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(Received November 16, 1995)

Abstract. We extend the theory of viscosity solutions for singular parabolic equations including, for example, axisymmetrized level set equation for mean curvature flow equation. We establish a comparison principle for viscosity solutions of singular degenerate parabolic equations including such an equation. We discuss the relation between axisymmetric viscosity solutions of original level set equation for mean curvature flow equation and the viscosity solution of axisymmetrized one.

Key words: singular degenerate parabolic equations, viscosity solutions, comparison principle, axisymmetric solutions.

1. Introduction

We are concerned with a degenerate parabolic equation of form:

$$u_t + F_0(\nabla_{x,r}u, \nabla^2_{x,r}u) - \frac{\nu u_r}{r^\beta} = 0 \ \text{ in } \ Q = (0,T)\times\Omega\times(0,R), \quad (1.1)$$

$$-u_r = 0 \ \text{ on } \ S = (0,T)\times\Omega\times\{0\}, \quad (1.2)$$

where Ω is a domain in $\boldsymbol{R}^m$, T, R and ν are positive numbers and β is a positive parameter. Here $u_t = \partial u/\partial t, \nabla_x u$ and $u_r = \partial u/\partial r$ denote the time derivative of u, the gradient of u in space variables x and the space derivative of u in r, respectively. We denote by $\nabla_{x,r}u = (\nabla_x u, u_r)$ and $\nabla^2_{x,r}u$ the gradient of u and the Hessian of u in space variables (x,r), respectively. The function $F_0 = F_0(p,X)$ is not continuous on $p=0$. As explained later in section 3, the equation (1.1) has many examples. One of them is of the form:

$$u_t - |\nabla_{x,r}u|\mathrm{div}_{x,r}\left(\frac{\nabla_{x,r}u}{|\nabla_{x,r}u|}\right) - \frac{n-m-1}{r}u_r = 0 \quad (1.3)$$

which is introduced as axisymmetrized level set equation for mean curvature flow equation, where for C^1 function $f_i : Q \to \boldsymbol{R}$ $(i = 1,\ldots,m+1)$ the

1991 Mathematics Subject Classification : 35K65, 35B05.