Stability of optical caustics with *r*-corners

Takaharu TSUKADA

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Abstract. In this papar, we investigate the stability of the optical caustic generated by a light source hypersurface with an r-corner in a smooth manifold under a fixed Hamiltonian system. Main results are the stability of optical caustics under the perturbation of hypersurfaces and a realization of a caustic as a stable optical caustic generated by some hypersurfaces.

Key words: lagrangian singularity, caustic, singularity.

1. Introduction

In [5] K. Jänich explained the wavefront propagation mechanism on a manifold which is completely described by a positive and positively homogeneous Hamiltonian function on the cotangent bundle and investigated the local gradient models given by the ray length function. He considered the case when the initial wave front is a smooth hypersurface without boundary. This case is corresponding to the theory of Lagrangian singularities (cf., [1]).

In this paper we consider the case when the initial wave front is a hypersurface with an r-corner (§1). The rays incident to conormal directions from each edges of the hypersurface gives a regular r-cubic configuration (cf., Section 3) at a point in the cotangent bundle, which is a generalized notion of Lagrangian submanifolds. The optical caustic with an r-corner generated by the hypersurface is given as the caustic of the regular r-cubic configuration. The notion of regular r-cubic configurations in complex analytic category has been introduced in [3], [4] and the real version has been developed in [8]. In [8] we have shown that any regular r-cubic configuration (at least locally) has a generating family which is a kind of families of functions. We also have shown that the stability of regular r-cubic configuration corresponds to the stable generating family.

In this paper we consider the following problems, extending of the investigations by K. Jänich [5] and G. Wassermann [10]: For a fixed Hamiltonian

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