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On the basis of twisted de Rham cohomology

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Abstract. The study of logarithmic form is essential to compute the cohomology group. First we will show the condition to represent (homogeneous) logarithmic $(n-1)$-forms by the logarithmic forms of type $\frac{dP}{P}$. By using this result, we can choose a basis for the twisted rational de Rham cohomology.

Key words: logarithmic forms.

Introduction

Let $D_j \subset \mathbb{C}^n$, $1 \leq j \leq m$ be a divisor defined by a polynomial $P_j(u) \in \mathbb{C}[u_1, \ldots, u_n]$, $1 \leq j \leq m$, and set D as the union of these. Define a covariant derivative

$$\nabla_\omega = d + \sum_{j=1}^{m} \frac{dP_j}{P_j} \wedge$$

on $M := \mathbb{C}^n - D$. The kernel of ∇_ω is the set of ∇_ω-horizontal sections. It defines a rank one local system $\mathcal{L}_\omega$. A general theory [KN] provided a nice interpretation of several integral representation of special function by means of duality between de Rham cohomology of ∇_ω and certain twisted cycles. In case that $P_j(u)$, $1 \leq j \leq m$, are all linear and in general position, [A1], [K] give beautiful representation of a basis for the top-dimensional de Rham cohomology group by logarithmic forms. The purpose of this article is to extend this study to our setting. The goal of this paper is Theorem 7.3.1 which gives a method to find an explicit basis of the top-dimensional twisted de Rham cohomology group. This is a natural generalization of a theorem of [K]. In this paper, we employ the condition Assumption 1.1.1 on $\bar{D} := \{\bar{P}_1 \cdots \bar{P}_m = 0\}$ and Assumption 1.2.1 on D as in [KN]. It is shown in [K] that there exists a gap, which is essential to our study, between the space of Saito's logarithmic forms $\Omega^p(\log D)$ and the space of ordinary logarithmic forms $\Omega^p\langle\mathcal{D}\rangle$. Let $\Omega^p(*D)$ be the space of rational p-forms with poles along D. The Grothendiek-Deligne comparison theorem asserts that there exists

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