Hokkaido Mathematical Journal Vol. 27 (1998) p. 423–455

Periodic points of disk homeomorphisms having a pseudo-Anosov component

Takashi MATSUOKA

(Received March 17, 1997; Revised June 30, 1997)

Abstract. We consider an orientation-preserving homeomorphism f of the disk. The braid type is one of the topological invariants defined for finite invariant sets of f. We show that if f has a finite invariant set S whose braid type contains a pseudo-Anosov component, then there are at least $2n+3$ periodic points of period $\leq n$ for any n with $n \geq \mathrm{per}(S)$, where $\mathrm{per}(S)$ is the maximum of the periods of the points in S.

Key words: braid, periodic point, pseudo-Anosov homeomorphism, disk homeomorphism.

1. Introduction

Let M be a compact connected orientable surface possibly with boundary, and $f : M \to M$ an orientation-preserving homeomorphism. Given a finite set S which is contained in the interior of M and is invariant under the map f, its braid type $\mathrm{bt}(S, f)$ is defined as the isotopy class of f relative to the set S [3] (see Section 2 below). From the Nielsen-Thurston classification theory of isotopy classes for surface homeomorphisms [6, 22], braid types are classified into three classes: pseudo-Anosov, periodic, and reducible. Moreover, in the reducible case, any braid type is decomposed into a finite number of irreducible components, each of which is either pseudo-Anosov or periodic.

This classification for braid types has great significance in the theory of 2-dimensional dynamical systems: If f has a finite invariant set S whose braid type $\mathrm{bt}(S, f)$ contains a pseudo-Anosov component, then f must have dynamical complexity, e.g. positive topological entropy and an infinite number of periodic points. Also, the following results have been obtained on the existence and the number of periodic points: The growth rate of the number of periodic points of period n, as n tends to infinity, is positive. (This follows from Jiang [16, Theorem 3.8]. See also [15].) There exist infinitely many period doubling sequences of periodic orbits (Guaschi [12, Theorem 4]). f has a periodic point with any sufficiently large period, provided that

1991 Mathematics Subject Classification : 58F20.