Hokkaido Mathematical Journal Vol. 27 (1998) p. 233–243

Algebraic descriptions of non-isolated singularities

Guangfeng JIANG*

(Received December 13, 1996; Revised July 9, 1997)

Abstract. For isolated singularities, there exist some algebraic characterizations called Mather-Yau statements. In this article, we generalize these to non-isolated singularities.

Key words: non-isolated singularity, $\mathcal{R}_I$-equivalence, right-left equivalence, isomorphism of algebras.

1. Introduction

Many authors have been trying to characterize singularities algebraically. Benson [B] and independently Shoshitaishvili [Sh2] have proved that, for (weighted) homogeneous hypersurface with isolated singularity, the Jacobian ideal of the defining polynomial determined completely its analytic equivalence class. Mather and Yau [MY] have proved that the moduli algebra of a hypersurface determined its analytic equivalence class. Scherk [Sc] and Yau [Y] have considered the $\mathcal{O}_1$ and respectively, $\mathbb{C}\{t\}/(t^{n+1})$-algebra structures on the Jacobian algebra $\frac{\mathcal{O}}{J(f)}$, and proved that this algebra determined completely the right-left equivalence class of function f with isolated singularities. This result has been generalized to functions on analytic varieties with isolated singularities by Matsuoka [M]. Dimca [Di] has considered whether the singular subspace of a complete intersection with isolated singularity can determine the analytic equivalence class of the whole space. Gaffney and Hauser [GH] and later Hauser and Müller [HM] have considered the singularities with isolated singularity type and so called harmonic singularities. For these singularities, the singular subspace, which may be non-isolated, determined completely the singularities. Martin [Ma] also gave some cohomology characterizations for some singularities.

We consider, in this paper, mainly non-isolated singularities. We find that some isomorphism between the ideals of algebras related to singularities can be lifted to an isomorphism between the algebras.

1991 Mathematics Subject Classification : 14B05, 32S05.

*Supported by National Nature Science Foundation of China, Liaoning Education Commission Science Foundation of China, and Utrecht Fellowship of The Netherlands