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Sublinear operators with rough kernel on generalized Morrey spaces

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Abstract. In this paper, we establish the boundedness of rough operators and their commutators with BMO functions in generalized Morrey spaces.

Key words: sublinear operator, Morrey space, commutator, BMO function.

The classical Morrey spaces were introduced in [6] by Morrey to study the local behaviour of solutions to second order elliptic partial differential equations. Since then, these spaces play an important role in studying the regularity of solutions to partial differential equations; see [2, 3]. In [5], Mizuhara introduced the following generalized Morrey spaces and discussed the boundedness of Calderón-Zygmund operators on these spaces.

Let ϕ be a positively growth function on $(0, \infty)$ and satisfy that for all $r > 0$,

$$\phi(2r) \leq D\phi(r), \tag{1}$$

where $D \geq 1$ is a constant independent of r.

Definition ([5]) Let $1 \leq p < \infty$. We denote by $L^{p,\phi} = L^{p,\phi}(\mathbf{R}^n)$ the space of locally integrable function f for which,

$$\int_{B_r(x_0)} |f(x)|^p \, dx \leq C^p \phi(r) \tag{2}$$

for all $x_0 \in \mathbf{R}^n$ and every $r > 0$, where $B_r(x_0) = \{x \in \mathbf{R}^n : |x - x_0| \leq r\}$; and we denote the smallest constant C satisfying (2) by $\|f\|_{L^{p,\phi}}$.

Obviously, when $\phi(r) = r^\lambda$, $0 < \lambda < n$, $L^{p,\phi}$ is just the classical Morrey spaces in [6].

The purpose of this paper is to establish the boundedness on the generalized Morrey spaces for a large class of sublinear operators with rough

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