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L^2-boundedness of Marcinkiewicz integral with rough kernel

Yong DING

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Abstract. In this paper the author given the L^2-boundedness of the Marcinkiewicz integral with rough kernel on product domains when Ω is in $L(\log^+ L)^2(S^{n-1}\times S^{m-1})$.

Key words: Marcinkiewicz integral, rough kernel, product domains.

1. Introduction

It is well known that in [6] E.M. Stein introducted the Marcinkiewicz integral operators of higher dimension as the following

$$\mu_\Omega(f)(x)=\left(\int_0^\infty |F_t(x)|^2\frac{dt}{t^3}\right)^{1/2},$$

where

$$F_t(x)=\int_{|x-y|\le t}\frac{\Omega(x-y)}{|x-y|^{n-1}}f(y)\,dy,$$

$\Omega\in L^1(S^{n-1})$ is a homogeneous of degree zero satisfying $\int_{S^{n-1}}\Omega(x')\,dx'=0$, and S^{n-1} denotes the unit sphere of $\mathbb{R}^n$. Stein proved that

Theorem A *Under the conditions above, if* $\Omega\in Lip_\alpha(S^{n-1})$ $(0<\alpha\le 1)$, *then*

$$\|\mu_\Omega(f)\|_p\le C\|f\|_p,\quad 1<p\le 2,$$

$$|\{x:x\in\mathbb{R}^n,\mu_\Omega(f)(x)>\lambda\}|\le\frac{C}{\lambda}\|f\|_1.$$

In [1], A. Benedek, A. Calderon and R. Panzone proved that if $\Omega\in C^1(S^{n-1})$, then $\mu_\Omega(f)$ is bounded operator on $L^p(\mathbb{R}^n)$ $(1<p<\infty)$. A. Torchinsky and S. Wang considered the weighted L^p-boundedness of

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