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Examples of global attractors in parabolic problems

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Abstract. Based on the theory of *dissipative systems* (see [HA]), necessary and sufficient conditions for the existence of global attractors for semilinear parabolic problems are studied. Many examples are considered to show precisely how this conditions works. We deal in particular with the Hodgkin-Huxley, Fitzhugh-Nagumo and Lotka-Volterra systems of reaction-diffusion equations, 2-D Navier-Stokes and Burgers equations of hydrodynamics and the Cahn-Hilliard pattern formation equation.

Key words: parabolic systems, global solutions, dissipative semigroups, global attractors.

1. Introduction

In this paper we consider an initial boundary value problem for an autonomous semilinear parabolic system of the form:

$$\begin{cases} u_t = -\mathcal{A}u + f(x, u, D^1u, \dots, D^k u), \ t > 0, \ x \in \Omega \subset R^n, \\ \mathcal{B}_j u = 0, \ t > 0, \ x \in \partial\Omega, \ j = 1, \dots, md, \\ u(0, x) = u_0(x), \ x \in \Omega. \end{cases} \tag{1}$$

Both $\mathcal{A} := (-1)^m \sum_{|\alpha| \le 2m} [a_\alpha^{rs}(x)]_{d\times d} D^\alpha$ and $\mathcal{B}_j := \sum_{|\beta| \le m_j} [b_\beta^{rs}(x)]_{1\times d} D^\beta$ $(j = 1, \dots, md)$ are linear matrix differential operators acting on $u = (u_1, \dots, u_d) : R^+ \times \Omega \to R^d$, Ω is a bounded domain in R^n with a regular boundary $\partial\Omega \in C^{2m}$, $k \le 2m - 1$ is a fixed nonnegative integer and D^j $(j = 1, \dots, k)$ denotes a vector of partial derivatives with respect to x of order j, i.e.

$$D^j u = \{D^\sigma u_1, \dots, D^\sigma u_d\}_{|\sigma| = j}.$$

We assume that:

A-I The triple $(\mathcal{A}, \{\mathcal{B}_j\}, \Omega)$ forms a *regular parabolic initial boundary value problem* of $2m$-th order in the sense of [AM, Chapt. 1, Sec. 1].

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