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Morita-Mumford classes on finite cyclic subgroups of the mapping class group of closed surfaces

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Abstract. Let G be a finite cyclic subgroup of the mapping class group of order m. We prove the Morita-Mumford classes restricted to G admit a certain kind of periodicity whose period is given by the Euler function $\phi(m)$. Using this periodicity theorem, we compute the Morita-Mumford classes on arbitrary finite cyclic subgroups of the automorphism group of Klein's quartic curve.

Key words: Morita-Mumford class, mapping class group, Klein curve.

Introduction

Let Σ_g be a closed oriented surface of genus $g \geq 2$, and M_g the mapping class group of Σ_g, which is the group of isotopy classes of orientation preserving diffeomorphisms of Σ_g. The cohomological study of M_g has been developed rapidly and has yielded many interesting results. The Morita-Mumford classes, defined by Morita [Mo1] and Mumford [Mu] independently, are a series of cohomology classes of M_g, whose zeroth term is equal to the Euler number $2-2g$ of Σ_g. Many mathematicians, including Harer [H2] [H3], Miller [Mi], and Morita [Mo1] [Mo2] [Mo3] [Mo4], have pointed out the importance of these classes for the study of the stable cohomology ring of M_g. Moreover, recently it is revealed by Akita that the Morita-Mumford classes play an important role in the study of the η-invariant of mapping tori of periodic mapping classes (see [Ak]). We are convinced that the Morita-Mumford classes contribute largely to the unstable cohomological study of M_g in the future.

The Morita-Mumford classes of surface bundles are defined as follows. Let $\pi : E \rightarrow B$ be an oriented fiber bundle whose fiber is Σ_g. (We call such a bundle a "surface bundle") The relative tangent bundle $T_{E/B}$ is the oriented real 2-dimensional vector bundle over E consisting of all the tangent vectors along the fibers. Take its Euler class $e := e(T_{E/B}) \in H^2(E; \boldsymbol{Z})$, then $e^{n+1} \in H^{2(n+1)}(E; \boldsymbol{Z})$. Let $\pi_! : H^n(E; \boldsymbol{Z}) \rightarrow H^{n-2}(B; \boldsymbol{Z})$ be the

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