Hokkaido Mathematical Journal Vol. 28 (1999) p. 435–460

Singular integrals with rough kernels on product spaces

Dashan FAN, Kanghui GUO and Yibiao PAN*

(Received March 4, 1998; Revised October 23, 1998)

Abstract. Suppose that $\Omega(x', y') \in L^1(S^{n-1} \times S^{m-1})$ is a homogeneous function of degree zero satisfying the mean zero property (1.1), and that $h(s, t)$ is a bounded function on $\mathbb{R}\times\mathbb{R}$. The singular integral operator Tf on the product space $\mathbb{R}^n\times\mathbb{R}^m$ $(n \geq 2, m \geq 2)$ is defined by

$$Tf(\xi, \eta) = \text{p.v.} \int_{\mathbb{R}^n\times\mathbb{R}^m} h(|x|, |y|)|x|^{-n}|y|^{-m}\Omega(x', y')f(\xi - x, \eta - y)dx\,dy.$$

We prove that the operator Tf is bounded in $L^p(\mathbb{R}^n \times \mathbb{R}^m)$, $p \in (1, \infty)$, provided that Ω is a function in certain block space $B_q^{0,1}(S^{n-1} \times S^{m-1})$ for some $q > 1$. The result answers a question posed in [JL].

We also study singular integral operators along certain surfaces.

Key words: singular integrals, rough kernel, block spaces, product spaces.

1. Introduction

Let $\mathbb{R}^N$ $(N = n$ or $m)$, $N \geq 2$, be the N-dimensional Euclidean space and S^{N-1} be the unit sphere in $\mathbb{R}^N$ equipped with normalized Lebesgue measure $d\sigma = d\sigma(\cdot)$. For nonzero points $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, we define $x' = x/|x|$ and $y' = y/|y|$. For $n \geq 2$, $m \geq 2$, let $\Omega(x', y') \in L^1(S^{n-1}\times S^{m-1})$ be a homogeneous function of degree zero, and satisfy

$$\int_{S^{n-1}} \Omega(x', y')d\sigma(x') = \int_{S^{m-1}} \Omega(x', y')d\sigma(y') = 0. \tag{1.1}$$

Let $h(s, t)$ be a locally integrable function on $\mathbb{R} \times \mathbb{R}$. The singular integral operator Tf on the product space $\mathbb{R}^n \times \mathbb{R}^m$ is defined by

$$(Tf)(x, y) = \text{p.v.} \int_{\mathbb{R}^n\times\mathbb{R}^m} K(\xi, \eta)f(x - \xi, y - \eta)d\xi\,d\eta \tag{1.2}$$

where $K(x, y) = h(|x|, |y|)\Omega(x', y')|x|^{-n}|y|^{-m}$ and f is a test function in $\mathcal{S}(\mathbb{R}^n \times \mathbb{R}^m)$. If $h = 1$ and Ω satisfies some regularity conditions, then it is known that the operator T is bounded in $L^p(\mathbb{R}^n \times \mathbb{R}^m)$, $1 < p < \infty$ (see [Fe]). That the L^p-boundedness of T continues to hold under the weaker

1991 Mathematics Subject Classification : Primary 42B20; Secondary 42B25.

*Supported in part by NSF Grant DMS-9622979