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On the diameter of closed minimal submanifolds in a real projective space

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Abstract. In this note, we prove an optimal lower bound estimate for the diameter of closed minimal submanifolds in a real projective space.

Key words: diameter, minimal submanifolds, real projective space.

1. Introduction

Many results for the *pinching problem* of closed minimal submanifolds in a rank one symmetric space have been obtained in the past years. One can find various *curvature pinching theorems* about them (cf. [2], [3], [6], [8]). In [1], Chen proved an optimal *volume pinching theorem* for the above minimal submanifolds. To author's knowledge, few is known about the *diameter pinching problem* of the same kind of minimal submanifolds. In this paper, we obtain an optimal lower bound for the diameter of closed minimal submanifolds in a real projective space.

Theorem 1 *Let M^n be an n-dimensional connected immersed closed minimal submanifold in $RP^m(1)$, the m-dimensional real projective space of curvature* 1. *Then the diameter of M^n satisfies $d(M^n) \geq \frac{\pi}{2}$ with equality holding if and only if M^n is totally geodesic.*

2. A Proof of Theorem 1

Before proving Theorem 1, we list the following

Lemma 1 [4] *An immersed closed minimal submanifold in a Riemannian manifold N of positive sectional curvature must intersect every closed totally geodesic hypersurface of N.*

Proof of Theorem 1. Let p be an arbitrary fixed point of M^n and denote by $RP_p^{m-1}(1)$ the closd totally geodesic hypersurface of $RP^m(1)$ which is

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