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Some operators on Lorentz spaces

Enji SATO

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Abstract. It is shown that the spaces $A(p,q)$ and $M(p,q)$ defined by Chen and Lai [1] coincide for $1<p<2$ and $1<q<\infty$. Also the Banach algebraic properties of Lorentz-improving operators are investigated.

Key words: Lorentz space, Fourier-Stietjes transform, operating function.

1. Introduction

Let G be a locally compact abelian group (LCA group), $dx=dm$ the Haar measure of G, and Γ the dual group. Also the space of bounded regular Borel measures on G will be denoted by $M(G)$, and $L^p(G)$ the L^p space with the norm $\|\cdot\|_p$ on G.

In this paper, we study the properties of some bounded linear operators on Lorentz spaces $L(p,q)$ $(=L(p,q)(G))$ $(1\leq p,\ q\leq\infty)$.

First we recall some definitions and basic properties of Lorentz spaces.

Definition 1.1 Let f be a complex-valued measurable function on G which is finite m a.e. The distribution function of f is defined by

$$m_f(y)=m\{x\in G\mid |f(x)|>y\}\quad (y\geq 0).$$

The non-increasing rearrangement of f is the function f^* defined by

$$f^*(t)=\inf\{y>0\mid m_f(y)\leq t\}\quad (t\geq 0).$$

The Lorentz space $L(p,q)$ is defined as the set of equivalence classes of functions f as above such that $\|f\|_{pq}^*<\infty$, where

$$\|f\|_{pq}^*=\begin{cases}\left(\dfrac{q}{p}\displaystyle\int_0^\infty (t^{1/p}f^*(t))^q\frac{dt}{t}\right)^{1/q} & \text{if } 1\leq p,\ q<\infty\\ \displaystyle\sup_{t\in(0,\infty)}\ t^{1/p}f^*(t) & \text{if } 1\leq p\leq\infty,\ q=\infty.\end{cases}$$

Since f^* and f have the same distribution function, it follows that

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