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A singular integral operator related to block spaces

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Abstract. Let $h(t)$ be an L^{∞} function on $(0,\infty)$, $\Omega(y')$ be a $B_q^{0,0}$ function on the unit sphere satisfying the mean zero property (1.1) and $P_N(t)$ be a real polynomial on $\mathbf{R}$ of degree N satisfying $P_N(0)=0$. We prove that the singular integral operator

$$\left(T_{P_N,h}f\right)(x)=p.v.\int_{\mathbf{R}^n} h(|y|)\Omega(y')|y|^{-n}f(x-P_N(|y|)y')dy$$

is bounded in $L^p(\mathbf{R}^n)$ for $1<p<\infty$, and the bound is independent of the coefficients of $P_N(t)$.

Key words: singular integral, rough kernel, block spaces.

1. Introduction

Let $\mathbf{R}^n$, $n \geqslant 2$, be the n-dimensional Euclidean space and $\mathbf{S}^{n-1}$ be the unit sphere in $\mathbf{R}^n$ equipped with the normalized Lebesque measure $d\sigma = d\sigma(x')$. Let $\Omega(x)$ be a homogenous function of degree zero, with $\Omega \in L^1(S^{n-1})$ and

$$\int_{\mathbf{S}^{n-1}} \Omega(x')d\sigma(x')=0, \tag{1.1}$$

where $x'=\frac{x}{|x|}$ for any $x\neq 0$.

Suppose that $h(t)\in L^{\infty}(0,\infty)$. Let $P_N(t)$ be a polynomial of degree N satisfying $P_N(0)=0$.

The singular integral operator $T_{P_N,h}f$ is defined by

$$\left(T_{P_N,h}f\right)(x)=p.v.\int_{\mathbf{R}^n} K(y)f(x-P_N(|y|)y')dy$$

where $y'=\frac{y}{|y|}\in \mathbf{S}^{n-1}$, $K(y)=h(|y|)\Omega(y')|y|^{-n}$ and $f\in S(R^n)$.

We denote $T_{P_N,h}$ by $T_{I,h}$ if $P_N(t)=t$; and we denote $T_{P_N,h}$ by T_I if $P_N(t)=t$ and $h(t)\equiv 1$.

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