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Note on C^{∞} functions with the zero property

(Dedicated to the memory of Etsuo Yoshinaga (1946–1995))

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Abstract. Suppose that all of C^{∞} functions $f_1, \ldots, f_k$ have the zero property. We give a necessary and sufficient condition for their product to have the same property. This is a generalization of Bochnak's result ([1]).

Key words: zero property, theorem of zeros.

1. Introduction

The theorem of zeros for ideals of C^{∞} functions was studied by J. Bochnak and J.J. Risler in the 1970's.

Let M be a connected manifold of class C^{∞} and J an ideal in the ring $C^{\infty}(M)$ of C^{∞} functions on M. We say that J *has the zero property* if all functions in $C^{\infty}(M)$ vanishing on the zeros of J belong to J. Also, we say that $f \in C^{\infty}(M)$ *has the zero property* if the principal ideal (f) has the zero property.

J. Bochnak shows that for an ideal J in $C^{\infty}(M)$ generated by a finite number of real analytic functions, J has the zero property if and only if J is real ([1]). He conjectures that for a finitely generated ideal J in $C^{\infty}(M)$, J has the zero property if and only if J is real and closed with respect to C^{∞} topology ([1]).

J.J. Risler shows that for a finitely generated ideal J in $C^{\infty}(\mathbb{R}^2)$, J has the zero property if and only if J is real and closed ([3]). Moreover for $f \in C^{\infty}(\mathbb{R}^3)$, he shows that if (f) is real and closed and the zero set of f satisfies a certain condition then f has the zero property ([3]). It is still an open problem to give a complete characterization of those finitely generated ideals of C^{∞} functions which have the zero property.

We are interested in the characterization of C^{∞} functions with the zero property. In this paper we treat the C^{∞} functions that can be expressed as a product of C^{∞} functions with the zero property. Namely, suppose that

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