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On some classes of regularization methods for minimization problem of quadratic functional on a half-space

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Abstract. This paper deals with some classes of a regularization methods of quadratic functional minimization problem on a half-space of real Hilbert space. We prove the convergence of the regularized solution. Under additional conditions, we obtain an estimate for convergence rate of the presented methods.

Key words: quadratic functional, stability, regularization.

1. Introduction

We consider the following extremal problem:

$$J(u) = \|Au - f\|^2 \to \inf, \quad u \in U = \{u \in H : \langle c, u\rangle \leq \beta\} \tag{1}$$

Here H and F are real Hilbert spaces; $A : H \to F$ is continuous linear operator; $f \in F$, $c \in H$, $c \neq 0$, are some fixed elements from the corresponding spaces; β is given real number.

In practice, instead of the exact operator A and the elements f, c, we deal with their approximations $A_\mu \in \mathcal{L}(H, F)$, $f_\delta \in F$, and $c_\sigma \in H$, such that

$$\|A - A_\mu\| \leq \mu, \ \ \|f - f_\delta\| \leq \delta, \ \ \|c - c_\sigma\| \leq \sigma,$$

where μ, δ and σ are small positive real numbers.

Generally speaking, the problem (1) is unstable with respect to the perturbations of the initial data A, f, c and the regularization method are required to solve it [3], [4], [5], [6], [7]. Remark that the regularization methods for the minimization problems

$$J(u) = \|Au - f\|^2 \to \inf, \quad u \in H \tag{2}$$

and

$$J(u) = \|Au - f\|^2 \to \inf, \quad u \in U = \{u \in H : \|u\| \leq R\}$$

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